11.23 Rhumb-line Sailing

The *sailings* in marine navigation refers to the various ways to figure a course and distance between two points on the globe, historically called "shaping a course." The related task is figuring a new Lat, Lon after sailing on a given course for a given distance. We might have thought that the course and distance from A to B would have a unique answer in navigation discussions, but it does not. The answers to these questions (although often very similar) depend on the type of sailing used. There are several solutions, each with pros and cons.

There are two general types of sailings: great-circle (GC) sailing, which yields the shortest distance between two points, and rhumb-line (RL) sailing, which yields a straight line on a Mercator chart (Section 11.4), the common map projection used on nautical charts. We cover GC sailing in Section 11.22. Commercial ships at higher latitudes often strive for GC routes, but it is not common in small-craft navigation, not to mention that sailing vessels must take the route the wind allows. As a rough guide, the difference between GC and RL solutions is only significant if *both* departure and arrival are at latitudes higher than about 45°, and the total distance is more than about 2,000 miles. Nevertheless, it can be valuable for a small-craft navigator to know how GC heading and distance compares to the RL solution when it comes to route selection, especially when you cannot steer the RL course for some reason.

Rhumb Line by Plotting

In one sense, the fundamental solution to RL sailings is just plot the two positions on a nautical chart and draw a straight line between them. After doing this, the course is easy to read relative to any meridian it crosses. The main virtue of a RL route is that the true heading remains constant along the full voyage—although we still have to correct for variation to get the magnetic heading, which will likely change over a long voyage.

For shorter routes, the RL distance between the two points is also easy to measure with dividers using the latitude scale of the chart. For longer distances, this is not so easy, because the chart scale (miles per inch) increases with the latitude on a Mercator chart. For moderate distances (whole route fits on a 1:80,000 chart) this can be accomplished by breaking the full route up into shorter latitude intervals and using corresponding scale for each latitude range.

For long route lengths, however, and especially those that cross from one chart to another, the plotted solution is not very convenient, and it grows less accurate in practice the longer the route. Because of this, a direct mathematical computation is often the best solution. With programmed calculators, mobile apps, or computer software available, the direct computation is the obvious solution. Fast and accurate.

Rhumb Line by Computation

There are two ways to solve RL sailings mathematically. You can use a shortcut method that is easy to solve with a trig calculator called *mid-latitude sailing* or you can compute a more generalized solution called *Mercator sailing*. This latter method is often done in conjunction with special tables found in *Bowditch* called Meridonal Parts, but it can be computed from scratch if needed. A software program or mobile app would almost certainly use the Mercator solution, because mid-latitude sailing is limited to route legs shorter than about 300 miles, especially for latitudes above 60° or so. Starpath.com/calc offers an online computation that can be used to compare the relative accuracies of these two methods.

Besides the mathematical approximations used, the two methods also differ in the shape of the earth they are based upon. Mid-latitude sailing assumes a spherical earth, whereas Mercator sailing incorporates the eccentricity of the earth, presumed flattened (to some very small extent) into the shape of a doorknob. With the computations matching the same earth shape used in the nautical chart datum, we get closer agreement with plotted values—if we had a way to plot to very high precision.

Alert to Reader!

Before proceeding, please note that computing RL sailing must be considered an advanced or specialized part of navigation training. This is true for two reasons. First, most calls for this underway or in planning can be done by plotting right on the chart, and second, if we are to use this formalism in the practical world, we should have a calculator or mobile app programmed for instant solutions. We include this here because USCG tests require this knowledge (without *programmed* devices) and we do work on the premise that we should be prepared to do *anything* we need on our own—assuming here that "on our own" includes using a trig calculator! We would also like to present our procedures for this, which differ in some important details from standard treatments.

Mid-latitude DR

Both RL sailing solutions used the same definitions as shown in Figure 11.23-1, because both are going to be straight lines on a Mercator chart. Furthermore, the latitude interval for a given run along a course will be the same for each solution. It is just how they compute the longitude interval over this run that changes.

First, we look at doing DR by mid-latitude sailing. That is, leaving from a Lat 1, Lon 1 and sailing for a distance D along course C, what is then your new Lat 2, Lon 2?

The definitions are illustrated in Figure 11.23-1. The first step in any sailing computation is to draw a small sketch that orients you for the solution. The importance of this step cannot be overstressed. In most cases we are computing intervals, and you must use your sketch to decide if the Lat and Lon are getting bigger or smaller based on the course direction.

Definitions:

D = distance run

C = true course

α = course angle

ℓ = Lat interval in nmi

dLat = Lat interval in degrees and minutes

p = *departure* = Lon interval in nmi

dLon = Lon interval in degrees and minutes

From trig we can compute the two sides of the right triangle:

$\ell = D \times \cos C$

$p = D \times \sin C$.

Since the latitude scale is the miles scale, we get directly

$dLat = \ell = D \times \cos C$.

Now we define the mid-latitude:

$Lm = (Lat\ 1 + Lat\ 2)/2$

Then we make the assumption that the meridians of longitude are getting closer as we leave the equator in proportion to the cos of the latitude, and choose the midpoint between the first and second positions to use for this.

$dLon = p/\cos Lm$

Then having computed the two angular intervals we can find the new values.

Lat 2 = Lat 1 + dLat

Lon 2 = Lon 1 + dLon

Note: Crossing the equator dLat = Lat 1+Lat 2. Crossing Greenwich, dLon = Lon 1+Lon 2, and crossing the Date Line, dLon = 360 - (Lon 1+Lon 2). This question should not be asked for distances much larger than 500 nmi outside of the tropics as the method becomes less accurate.

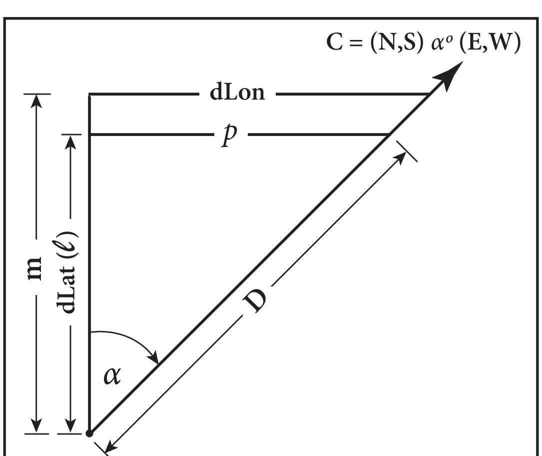

Figure 11.23-1. *Rhumb line sailing terms.*

Mid-Latitude Route

Now we look at the same solution applied to route planning. That is, given starting point (Lat 1, Lon 1) and destination (Lat 2, Lon 2) ,what is the distance between them and the course between them?

The procedure is to find ℓ and p, and then find C from

C = (N,S) α^o (E,W),

where

$\alpha = \arctan(p/\ell)$.

$p = dLon \times \cos Lm$

$\ell = dLat$

Then we can find D from Pythagorean Theorem:

$D^2 = \ell^2 + p^2$

$D = \text{sqrt}(\ell^2 + p^2)$.

Mid-Latitude Examples

Here are two questions from a USCG licence exam.

<u>DR by mid-latitude</u>

Example 1: A vessel steams 720 miles on course 058°T from LAT 30°06.0'S, LONG 31°42.0'E. What are the latitude and longitude of the point of arrival by mid-latitude sailing?

Solution: Make a rough sketch (Figure 11.23-3) for orientation and to ID dLat, dLon, course, distance run, and departure.

Lat 2 = Lat 1 + dLat.

dLat = D × cos C = 720 × cos (58) = 381.5' = 6° 21.5'.

So Lat 2 = 30° 06' - 6° 21.5' = 23° 44.5' S.

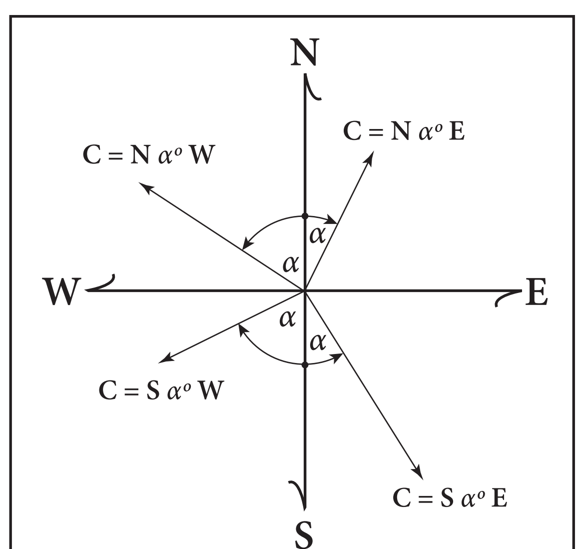

Figure 11.23-2. *How the angle α (called course angle) is used to determine the course direction.*

Lon 2 = Lon 1 - dLon

dLon = dep / cos (Lm)

p = D × sin C = 720 × sin (58) = 610.6'.

Lm = (23 44.5 + 30 06)/2 = (53 50.5)/2 = 26° 30' + 25.3' = 26 55.3' = 26.92°.

dLon = 610.6 / cos (26.92) = 684.8' = 11° 24.8'

Lon 2 = Lon 1 + dLon = 31°42.0' + 11° 24.8' = 42° 66.8' = 43° 6.8'E

Route by mid-latitude

Example 2: You depart LAT 28° 55.0'N, LONG 89° 10.0'W, enroute to LAT 24° 25.0'N, LONG 83° 00.0'W. Determine the true course and distance by mid-latitude sailing?

Solution: First make a rough sketch for orientation (Figure 11.23-4), roughly to scale, but precise scale is not crucial. From this we see that the course is given by C = S α E. Then find ℓ and p, and from these find C and D.

Lat 1 = 28° 55.0'N	Lon 1 = 89° 10.0'W
Lat 2 = 24° 25.0'N	Lon 2 = 83° 00.0'W
dLat = 4° 30.0' = 270.0'	dLon = 6° 10.0' = 370.0'

ℓ = dLat = 270.0'

Lm = (28° 55' + 24° 25')/2 = 52° 80' / 2 = 26° 40' = 26.67°

p = dLon × cos(Lm) = 370 × cos(26.67) = 370 × 0.89363 = 330.6 nmi

α = arctan(p/ℓ) = arctan(330.6/270.0) = 50.8°

Referring to Figure 11.23-4,

C = S α^o E, or 180 - 50.8 = 129.2°

D = sqrt (270.0^2 + 330.6^2) = 426.8 nmi.

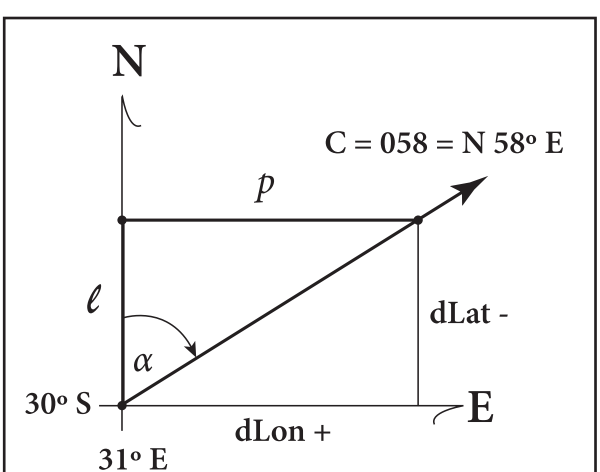

Figure 11.23-3. *Example 1 sketch to determine if dLat and dLon are plus or minus.*

Mercator Sailing

The Mercator sailing solution is a more accurate representation of a RL on a Mercator chart than mid-latitude sailing, especially for longer distances at higher latitudes. This is achieved by using directly the data that define the Mercator chart, namely the Meridonal Parts. The meridonal part (M) for a given latitude is the distance along a meridian from that latitude to the equator on a Mercator chart, expressed in minutes of longitude at the equator. This is just the data needed to construct an accurate Mercator chart to match a specific chart datum, usually WGS-84.

In mid-latitude sailing we just assumed there is 1 nmi per 1' of lat, which is no longer true on a non-shperical earth, and we approximated the distance between them (the departure) with the cos (Lat). Thus, we no longer use ℓ and p to find the course angle, but now use the difference between the meridonal parts (m) and dLon directly. A key to remembering the new equations is the fact that m has units of longitude minutes.

We still need to make the sketch to figure the course angle (α) and the directions for Lat and Lon increments.

Mercator Route

For a Mercator route between two positions, use:

α = arctan (dLon/m),

where m = M1 ± M2. This difference in meridonal parts is - when both positions are on the same side of the equator, but + when the route crosses the equator.

C = (N,S) α (E,W),

D = dLat/cos C,

The M values for Lat 1 and Lat 2 can be looked up in Table 6 of *Bowditch* (Figure 11.23-5, available online and always in the USCG test room), which lists them for every 1' of Lat, or compute directly from:

M(Lat) = 7915.704468 × log [tan (45° + Lat/2)] - 23.0133633 × sin (Lat) -0.051353 × sin ^{3} (Lat) - 0.000206 × sin^5 (Lat).

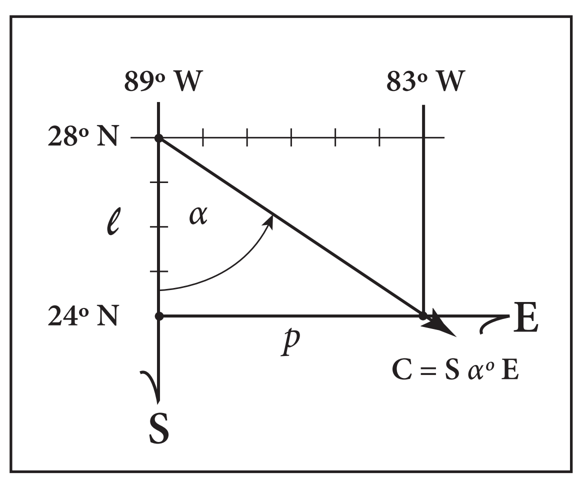

Figure 11.23-4. *Example 2 sketch of the example question to determine C from the course angle α.*